



Lancaster University 

PSYC214: Statistics
Lecture 4 – One-factor within-participants ANOVA – Part I

Michaelmas Term
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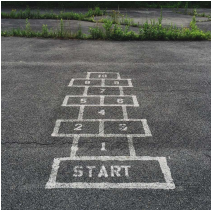
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One factor within-participants ANOVA Lancaster University 


Agenda/Content for Lecture 4


- Introduction to one factor within-participants ANOVA and its limitations
- Between-participant variability and residual variance
- Calculating within-group and between group variances
- Producing the within-participants F-statistic



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Between-participants Lancaster University 




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Within-participants Lancaster University 




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

Within-participants design - limitations Lancaster University 

Type	Definition	An example...
Practice effects	The experience/performance on a task at a given point in time, may influence your performance of that task at a subsequent time.	

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Within-participants design - limitations Lancaster University 

	Type	Definition	An example...
Order effects	Practice effects	The experience/performance on a task at a given point in time, may influence your performance of that task at a subsequent time.	
	Fatigue effects	Fatigue or boredom with a task may influence your performance of that task at a subsequent time.	

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Within-participants design - limitations 


Type	Definition	An example...
Order effects	Practice effects The experience/performance on a task at a given point in time, may influence your performance of that task at a subsequent time.	
	Fatigue effects Fatigue or boredom with a task may influence your performance of that task at a subsequent time.	
Demand characteristic	Participants form an idea of the experiment's purpose and (sub)consciously change their behaviour to comply	

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Assumptions underlying the W-P ANOVA 


1. Assumption of **independence**



Independence


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Assumptions underlying the W-P ANOVA 

1. Assumption of independence


2. Assumption of **normality**



Independence Normality


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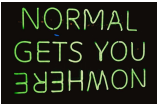
Assumptions underlying the W-P ANOVA 

1. Assumption of independence
2. Assumption of normality
3. Assumption of **sphericity**


The variances of the differences between all combinations of related groups are equal



Independence




Normality





Sphericity

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Between-participants F ratio 


$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$



$$F = \frac{\text{treatment effects} + \text{experimental error}}{\text{experimental error} + \text{individual differences} + \text{random (residual) errors}}$$

$$F = \frac{\text{treatment effects} + \text{individual differences} + \text{random (residual) errors}}{\text{individual differences} + \text{random (residual) errors}}$$

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Within-participants F ratio 

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{\text{treatment effects} + \text{random (residual) errors}}{\text{random (residual) errors}}$$

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The F ratio

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$F = \frac{\text{Signal}}{\text{Noise}}$

$F = \frac{\text{Signal}}{\text{Noise}}$

The larger in magnitude the F value, the more treatment effects are standing out away from experimental error – i.e., the larger the signal is from the noise. The larger the F, the less likely that differences in scores are caused by chance.

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A within-participants example

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A within-participants example

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	A ₁	A ₂	ΔA	P Mean
P ₁	3	1	-2	2
P ₂	5	3	-2	4
P ₃	4	2	-2	3
P ₄	5	3	-2	4
P ₅	5	3	-2	4
A Mean	4.4	2.4	-2	

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A within-participants example




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Table 2. Burgers consumed before (A1) and after (A2) Cowsspiracy

	A1	A2	ΔA	P Mean
P1	1	3	2	2
P2	3	5	2	4
P3	2	4	2	3
P4	3	5	2	4
P5	3	5	2	4
A Mean	2.4	4.4	2	

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A within-participants example




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Table 3. Burgers consumed before (A1) and after (A2) Cowsspiracy

	A1	A2	ΔA	P Mean
P1	3	1	-2	2
P2	5	4	-1	4.5
P3	4	1	-3	2.5
P4	5	1	-4	3
P5	5	3	-2	4
A Mean	4.4	2	-2.4	

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A within-participants example




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Table 4. Burgers consumed before (A1) and after (A2) Cowsspiracy

	A1	A2	ΔA	P Mean
P1	3	5	2	4
P2	5	4	-1	4.5
P3	4	5	1	4.5
P4	5	1	-4	3
P5	5	5	0	5
A Mean	4.4	4	-0.4	

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Residual variance


Table 5. Burgers consumed before (A_i) and after (A_i) Cowsspiracy

	A _i	A _i	ΔA _i	P Mean
P1	5	3	-2	4
P2	9	7	-2	8
P3	3	1	-2	2
P4	7	5	-2	6
P5	4	6	2	5
A Mean	5.6	4.4		5

High between-participant variability / Low residual variance

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Residual variance

Table 5. Burgers consumed before (A_i) and after (A_i) Cowsspiracy

	A _i	A _i	ΔA _i	P Mean
P1	5	3	-2	4
P2	9	7	-2	8
P3	3	1	-2	2
P4	7	5	-2	6
P5	4	6	2	5
A Mean	5.6	4.4		5


High between-participant variability / Low residual variance

The variability in the consistency of trends

- In this example, these trends overall are pretty consistent.
- [-2, -2, -2, -2, 2].
- Most are same direction and -2 in difference.
- As such, the residual variance is said to be low

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Residual variance

The variability in the consistency of trends

- In this example, there trends are very inconsistent.
- [-8, 0, 2, -2, 2] = widespread.
- As such, the residual variance is said to be high.


Low between-participant variability / High residual variance

Table 6. Burgers consumed before (A_i) and after (A_i) Cowsspiracy

	A _i	A _i	ΔA _i	P Mean
P1	9	1	-8	5
P2	5	5	0	5
P3	4	6	2	5
P4	6	4	-2	5
P5	4	6	2	5
A Mean	5.6	4.4		5

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Summary

Table 5. Burgers consumed before (A1) and after (A2) Conspiracy

	A1	A2	ΔA	P Mean
P1	5	3	-2	4
P2	9	7	-2	8
P3	3	1	-2	2
P4	7	5	-2	6
P5	4	6	2	5
A Mean	5.6	4.4		5

High between-participant variability / **Low** residual variance

Table 6. Burgers consumed before (A1) and after (A2) Conspiracy

	A1	A2	ΔA	P Mean
P1	9	1	-8	5
P2	5	5	0	5
P3	4	6	2	5
P4	6	4	-2	5
P5	4	6	2	5
A Mean	5.6	4.4		5


Low between-participant variability / **High** residual variance

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Within-participants F ratio

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$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{\text{between-group variance}}{\text{residual variance}}$$

We calculate the F ratio in a similar way as for the between participants design, with the exception that we are not interested in how participants vary from one another!

We therefore include an additional step to remove the between-participant variability (we spoke of before) from the error term.

We remove the between-participant variability from the within-group variability – leaving only random errors behind – a.k.a., the residual variability

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Ingredients of within-participants ANOVA

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Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$$F = \frac{\text{between-group variance}}{\text{residual variance}}$$

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Ingredients of within-participants ANOVA

Lancaster University

Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$$SS_{\text{BETWEEN}} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A} - \frac{(\Sigma Y)^2}{N}$$

$$SS_{\text{WITHIN}} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A}$$


$$SS_{\text{TOTAL}} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N}$$

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SS-Between groups

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$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A} - \frac{(\Sigma Y)^2}{N}$$


Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

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SS-Between groups

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$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A} - \frac{(\Sigma Y)^2}{N}$$

Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$$SS_{BETWEEN} = \frac{(20)^2 + (41)^2 + (48)^2}{9} - \frac{(109)^2}{27}$$

$$SS_{BETWEEN} = \frac{400 + 1681 + 2304}{9} - \frac{11881}{27}$$

$$SS_{BETWEEN} = 44.44 + 186.77 + 256.00 - 440.03$$

$$SS_{BETWEEN} = 487.21 - 440.03$$


$$SS_{BETWEEN} = 47.18$$

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Ingredients of within-participants ANOVA

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Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48


$$SS_{BETWEEN} = 47.18$$


$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A}$$

$$SS_{TOTAL} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N}$$

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Ingredients of within-participants ANOVA 



Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48


$$SS_{BETWEEN} = \frac{(\sum A_1)^2 + (\sum A_2)^2 + (\sum A_3)^2}{N_A} - \frac{(\sum Y)^2}{N}$$


$$SS_{WITHIN} = \sum Y^2 - \frac{(\sum A_1)^2 + (\sum A_2)^2 + (\sum A_3)^2}{N_A}$$

$$SS_{TOTAL} = \sum Y^2 - \frac{(\sum Y)^2}{N}$$

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SS-Total 



Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2 ² = 4	3 ² = 9	5 ² = 25
2	1 ² = 1	4 ² = 16	4 ² = 16
3	3 ² = 9	5 ² = 25	6 ² = 36
4	2 ² = 4	6 ² = 36	5 ² = 25
5	2 ² = 4	3 ² = 9	3 ² = 9
6	1 ² = 1	5 ² = 25	6 ² = 36
7	4 ² = 16	7 ² = 49	7 ² = 49
8	3 ² = 9	3 ² = 9	6 ² = 36
9	2 ² = 4	5 ² = 25	6 ² = 36
Total	20	41	48

$$SS_{TOTAL} = \sum Y^2 - \frac{(\sum Y)^2}{N}$$

$$SS_{TOTAL} = 523 - \frac{(109)^2}{27}$$


$$SS_{TOTAL} = 523 - \frac{11881}{27}$$


$$SS_{TOTAL} = 523 - 440.03$$

$$SS_{TOTAL} = 82.97$$

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Ingredients of within-participants ANOVA 



Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

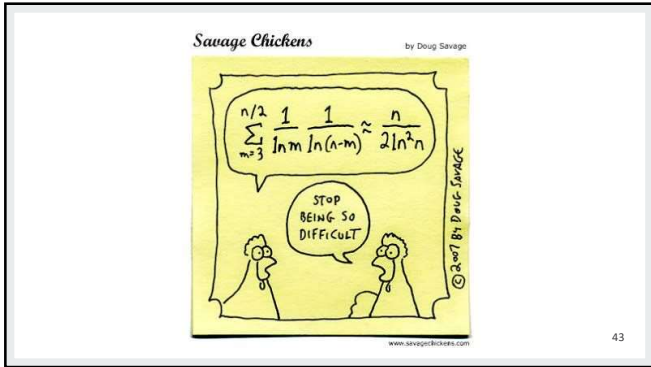
$$SS_{BETWEEN} = 47.18$$

$$SS_{WITHIN} = 35.79$$


$$SS_{TOTAL} = 82.97$$

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Ingredients of within-participants ANOVA Lancaster University 

Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$SS_{BETWEEN} = 47.18$

$SS_{WITHIN} = 35.79$

$SS_{TOTAL} = 82.97$

$$SS_{between\ participants} = \frac{(\sum P_1)^2 + (\sum P_2)^2 \text{ (and so on)}}{N_p} - \frac{(\sum Y)^2}{N}$$

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SS-between participants

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$$SS_{\text{between participants}} = \frac{(\sum P_1)^2 + (\sum P_2)^2 \text{ (and so on)}}{N_p} - \frac{(\sum Y)^2}{N}$$

Participant	A ₁ scores	A ₂ scores	A ₃ scores	P total
1	2	3	5	10
2	1	4	4	9
3	3	5	6	14
4	2	6	5	13
5	2	3	3	8
6	1	5	6	12
7	4	7	7	18
8	3	3	6	12
9	2	5	6	13
Total	20	41	48	109

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SS-between participants

Lancaster University

$$SS_{\text{between participants}} = \frac{(\sum P_1)^2 + (\sum P_2)^2 \text{ (and so on)}}{N_p} - \frac{(\sum Y)^2}{N}$$

Participant	A ₁ scores	A ₂ scores	A ₃ scores	P total
1	2	3	5	10
2	1	4	4	9
3	3	5	6	14
4	2	6	5	13
5	2	3	3	8
6	1	5	6	12
7	4	7	7	18
8	3	3	6	12
9	2	5	6	13
Total	20	41	48	109

$$\left(\frac{10^2}{3} + \frac{9^2}{3} + \frac{14^2}{3} + \frac{13^2}{3} + \frac{8^2}{3} + \frac{12^2}{3} + \frac{18^2}{3} + \frac{12^2}{3} + \frac{13^2}{3}\right) - \frac{(109)^2}{27}$$

$$\left(\frac{100 + 81 + 196 + 169 + 64 + 144 + 324 + 144 + 169}{3}\right) - \frac{(109)^2}{27}$$

$$(33.33 + 27 + 65.33 + 56.33 + 21.33 + 48 + 108 + 48 + 56.33) - 440.03$$

$$463.67 - 440.03 = 23.64$$

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Ingredients of within-participants ANOVA

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Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

SS_{BETWEEN} = 47.18

SS_{WITHIN} = 35.79


SS_{TOTAL} = 82.97

SS_{between participants} = 23.64

SS_{RESIDUAL}

48


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What we'll need for the ANOVA Lancaster University 

$$SS_{RESIDUAL} = SS_{WITHIN} - SS_{between\ participants}$$

$$12.15 = 35.79 - 23.64$$

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Ingredients of within-participants ANOVA Lancaster University 

Participant	A ₁ scores	A ₂ scores	A ₃ scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$SS_{BETWEEN} = 47.18$

$SS_{WITHIN} = 35.79$


$SS_{TOTAL} = 82.97$

$SS_{between\ participants} = 23.64$

$SS_{RESIDUAL} = 12.15$

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What we'll need for the ANOVA Lancaster University 


$$F = \frac{\text{between-group variance}}{\text{residual variance}}$$


$$\text{between-group variance} = \frac{SS_{BETWEEN}}{df_{BETWEEN}} = \frac{47.18}{2} = 23.59$$

↘ a - 1 [i.e., number of levels - 1]

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What we'll need for the ANOVA

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$$F = \frac{23.59}{0.76} = 31.04 \text{ WAY BIGGER THAN } 3.6337!$$



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Lecture 4 – One-factor within-participants ANOVA

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Review of lecture 4

- Introduction to one factor within-participants ANOVA and its limitations
- Between-participant variability and residual variance
- Calculating within-group and between group variances
- Producing the within-participants F-statistic



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Savage Chickens by Doug Savage

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